Complex powers, roots and polynomial factors

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A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Attempt the reasoning and exact calculations by hand. Technology may check a result but cannot replace the written argument.

What you will practise

• Complex numbers, De Moivre’s theorem and polynomial factorisation

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

Write a nonzero complex number as r cis θ, where cis θ=cos θ+i sin θ. De Moivre’s theorem gives $(r \operatorname{cis} \theta)^n = r^n \operatorname{cis}(n\theta)$ for integer n. Multiplication adds arguments and multiplies moduli.

The n distinct nth roots of a nonzero r cis θ have modulus $r^{(1/n)}$ and arguments $(\theta+2k\pi)/n$ for k=0, ..., n-1. The fundamental theorem of algebra guarantees a degree-n complex polynomial has n roots counting multiplicity.

Worked example

Find all cube roots of 8.

Use modulus 2 and arguments 0, 2π/3, 4π/3

convert each root to Cartesian form.

Answer: 2, -1+i√3, -1-i√3.

Try it yourself

Name: _____ Date: _____

1. Calculate $(1+i)^4$.

2. Find the fourth roots of 16.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. -4.

$1+i=\sqrt{2}\,\mathrm{cis}(\pi/4)$; the fourth power is $4\,\mathrm{cis}\,\pi$.

2. 2, 2 i, -2, -2 i.

Use modulus 2 and arguments $k\pi/2$ for $k=0, 1, 2, 3$.

Curriculum reference: Specialist Mathematics Units 3–4 — Complex numbers, De Moivre’s theorem and polynomial factorisation

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