Differential equations, logistic models and Euler’s method

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Show the mathematical steps. A calculator, graphing tool or CAS may be used to check results where useful.

What you will practise

• Differential equations, logistic growth, separation and Euler’s method

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

A differential equation connects a quantity with its rate of change. Initial conditions select a particular solution. For a separable equation $y' = f(x)g(y)$, separate the variables and integrate, while checking equilibrium solutions that division by $g(y)$ may exclude.

A logistic model $P' = r\,P(1-P/K)$ has equilibria 0 and K. Euler’s method approximates y at the next x value by $y_{n+1} = y_n + h\,f(x_n, y_n)$. Its straight-line steps can have substantial error when the step size is large.

Worked example

Solve $dy/dx = 2xy$ with $y(0) = 3$.

For $y \neq 0$, integrate $dy/y = 2x\,dx$ to obtain $\ln|y| = x^2 + C$

write $y = Ae^{(x^2)}$

the initial condition gives $A = 3$.

Answer: $y = 3e^{(x^2)}$.

Try it yourself

Name: _____ Date: _____

1. Solve $y' = 3x^2$ with $y(1) = 5$.

2. Solve $y' = -2y$ with $y(0) = 4$.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. $y=x^3+4$.

Integrate to x^3+C, then use $5=1+C$.

2. $y=4\ e^{(-2\ x)}$.

Separate variables or recognise exponential decay; the initial value fixes the multiplier.

Curriculum reference: Specialist Mathematics Units 3–4 — Differential equations, logistic growth, separation and Euler’s method

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