Implicit differentiation and related rates

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A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Attempt the reasoning and exact calculations by hand. Technology may check a result but cannot replace the written argument.

What you will practise

• Inverse circular derivatives, second derivatives, implicit differentiation and related rates

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

When y depends on x, differentiate an implicit relation using the chain rule on every y term. For example, $d(y^2)/dx=2\,y\,y'$. Collect the y' terms before solving. The formula may fail where its denominator vanishes.

Related rates link quantities changing with time. First state a geometric or physical relationship, then differentiate with respect to time and substitute the values at the requested instant. Inverse circular derivatives require their real domains.

Worked example

For $x^2+y^2=25$, find dy/dx at (3, 4).

Differentiate to obtain $2x+2\,y\,y'=0$

solve $y'=-x/y$

substitute (3, 4).

Answer: -3/4.

Try it yourself

Name: _____ Date: _____

1. Differentiate $y=\arcsin(2x)$ and give the derivative's domain.

2. Differentiate $y=\arctan(3x)$.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. $y' = 2/\sqrt{(1-4x^2)}$, for $-1/2 < x < 1/2$.

Apply the chain rule; the derivative denominator must be positive.

2. $3/(1+9x^2)$.

Use the arctangent derivative and inner derivative 3.

Curriculum reference: Specialist Mathematics Units 3–4 — Inverse circular derivatives, second derivatives, implicit differentiation and related rates

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