Area, arc length and solids of revolution

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Show the mathematical steps. A calculator, graphing tool or CAS may be used to check results where useful.

What you will practise

• Areas, parametric arc lengths, surface areas and volumes of revolution

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

Integration adds small geometric contributions. A region’s area is the integral of upper minus lower height. A volume from discs or washers is the integral of π times the difference of squared radii; the radius is measured to the stated axis.

For a smooth parametric curve, arc length is $\int\sqrt{[(dx/dt)^2+(dy/dt)^2]}\,dt$. Surface area on revolution about the x-axis is $\int 2\pi|y|\sqrt{[(dx/dt)^2+(dy/dt)^2]}\,dt$. State the parameter interval and check whether a curve retraces itself.

Worked example

The region under $y=x$ for $0\leq x\leq 2$ is revolved about the x-axis. Find its volume.

A cross-sectional disc has radius x and area πx^2

integrate πx^2 from 0 to 2.

Answer: $8\pi/3$ cubic units.

Try it yourself

Name: _____ Date: _____

1. Find the area between $y=x$ and $y=x^2$ for $0 \leq x \leq 1$.

2. Find the arc length of $x=3t$, $y=4t$ for $0 \leq t \leq 2$.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. 1/6 square unit.

The line is above the parabola; integrate $x-x^2$.

2. 10 units.

The integrand is $\sqrt{(3^2+4^2)}=5$; multiply by the parameter interval length 2.

Curriculum reference: Specialist Mathematics Units 3–4 — Areas, parametric arc lengths, surface areas and volumes of revolution

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