Advanced integration techniques

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Show the mathematical steps. A calculator, graphing tool or CAS may be used to check results where useful.

What you will practise

• Substitution, inverse circular forms, trigonometric identities, partial fractions and integration by parts

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

Choose the technique from the structure. An inner expression with its derivative suggests substitution; a product such as $x\,e^x$ suggests integration by parts; a rational function may simplify by division or partial fractions.

Useful forms include $\int 1/x\,dx=\ln|x|+C$, $\int 1/\sqrt{(a^2-x^2)}\,dx=\arcsin(x/a)+C$ and $\int a/(a^2+x^2)\,dx=\arctan(x/a)+C$ for $a>0$. Check an indefinite integral by differentiating and retain domain restrictions.

Worked example

Find $\int x\,e^x\,dx$.

Choose $u=x$ and $dv=e^x\,dx$

integration by parts gives $x\,e^x-\int e^x\,dx$

simplify.

Answer: $(x-1)e^x+C$.

Try it yourself

Name: _____ Date: _____

1. Find $\int 3/x \, dx$ for $x \neq 0$.

2. Find $\int 1/\sqrt{(9-x^2)} \, dx$ for $-3<x<3$.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. $3\ln|x|+C$.

Use the logarithmic antiderivative on each connected interval of the domain.

2. $\arcsin(x/3)+C$.

Recognise the inverse-sine derivative with $a=3$.

Curriculum reference: Specialist Mathematics Units 3–4 — Substitution, inverse circular forms, trigonometric identities, partial fractions and integration by parts

Independent original resource. Not published or endorsed by VCAA. This focused set supports the listed skills; it is not a complete course or an official assessment.

This free preview may be shared intact with attribution. Not for sale.