Linear combinations of random variables

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Show the mathematical steps. A calculator, graphing tool or CAS may be used to check results where useful.

What you will practise

• Distribution of linear combinations of random variables

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

Expected values are linear: $E(a X+b Y+c)=a E(X)+b E(Y)+c$. For independent X and Y, $Var(a X+b Y+c)=a^2 Var(X)+b^2 Var(Y)$. A constant changes the mean but does not add variability.

Independent normal variables have normal linear combinations. In this collection $N(\mu, \sigma^2)$ uses variance as its second parameter. A difference still adds the appropriately weighted variances when the variables are independent.

Worked example

Independent X and Y have means 10 and 4 and variances 9 and 1. Find the mean and variance of $2 X-3 Y+5$.

$Mean=2\times10-3\times4+5$

$variance=2^2\times9+(-3)^2\times1$.

Answer: Mean 13; variance 45.

Try it yourself

Name: _____ Date: _____

1. Five independent identically distributed quantities each have mean 8 and variance 4. Find the mean and variance of their sum.

2. X has mean 3 and variance 2. Find the mean and variance of 4 X-1.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. Mean 40; variance 20.

Add the five means and, by independence, the five variances.

2. Mean 11; variance 32.

Multiply the mean by 4 and subtract 1; multiply variance by 4^2.

Curriculum reference: Specialist Mathematics Units 3–4 — Distribution of linear combinations of random variables

Independent original resource. Not published or endorsed by VCAA. This focused set supports the listed skills; it is not a complete course or an official assessment.

This free preview may be shared intact with attribution. Not for sale.