Logic, proof and mathematical induction

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Attempt the reasoning and exact calculations by hand. Technology may check a result but cannot replace the written argument.

What you will practise

• Logic and proof

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

A proof establishes a statement for every case in its stated domain. An example can support a conjecture, but a single counterexample disproves a universal claim. Distinguish an implication from its converse and distinguish necessary conditions from sufficient conditions.

Induction requires a base case and a step from an arbitrary valid integer k to k+1. State the induction hypothesis precisely and use it only for the assumed case; checking several numerical cases is not the induction step.

Worked example

Prove by induction that $1+3+\ldots+(2\,n-1)=n^2$ for every positive integer n.

Base n=1 gives $1=1^2$

assume the sum of the first k odd numbers is k^2

adding 2 k+1 gives $k^2+2\,k+1=(k+1)^2$

induction proves the claim.

Answer: The identity holds for every integer $n\geq 1$.

Try it yourself

Name: _____ Date: _____

1. Negate: for every real x, $x^2 > x$.

2. For integers n, prove that if n^2 is even, then n is even.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. There exists a real x such that $x^2 \leq x$.

Negate the universal quantifier and negate the inequality. The value x=0 is a counterexample to the original statement.

2. Use the contrapositive: odd n has an odd square.

Write $n=2k+1$; $n^2=4k^2+4k+1=2(2k^2+2k)+1$, which is odd.

Curriculum reference: Specialist Mathematics Units 3–4 — Logic and proof

Independent original resource. Not published or endorsed by VCAA. This focused set supports the listed skills; it is not a complete course or an official assessment.

This free preview may be shared intact with attribution. Not for sale.