Rational and quotient graphs

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Show the mathematical steps. A calculator, graphing tool or CAS may be used to check results where useful.

What you will practise

- Rational functions and simple quotient functions

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

Find the domain before simplifying a quotient. A cancelled factor may create a hole rather than a vertical asymptote; preserve the original exclusion. Polynomial division identifies oblique asymptotes when the numerator has one higher degree than the denominator.

A complete curve analysis combines intercepts, asymptotes, symmetry, derivatives and end behaviour. Stationary points satisfy $f'=0$ inside the domain; an inflection point requires a change in concavity at a point on the curve.

Worked example

Analyse $y=(x^2+1)/x$: give its domain, asymptotes and stationary points.

Rewrite $y=x+1/x$

$y'=1-1/x^2$ vanishes at $x=\pm1$

$y''=2/x^3$ gives the stated classifications.

Answer: Domain $x\neq0$; asymptotes $x=0$ and $y=x$; local maximum (-1, -2), local minimum (1, 2).

Try it yourself

Name: _____ Date: _____

1. Simplify $(x^2-1)/(x-1)$, retaining its domain.

2. Find the vertical and horizontal asymptotes of $(2x+1)/(x-3)$.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. $y=x+1$ for $x\neq 1$, with a hole at $(1, 2)$.

Factor the numerator as $(x-1)(x+1)$; cancelling does not restore the excluded input.

2. $x=3$ and $y=2$.

The denominator vanishes at 3 while the numerator does not. Equal degrees give the leading-coefficient ratio 2.

Curriculum reference: Specialist Mathematics Units 3–4 — Rational functions and simple quotient functions

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