Sampling distributions and the central limit theorem

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A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Show the mathematical steps. A calculator, graphing tool or CAS may be used to check results where useful.

What you will practise

• Distribution of the sample mean

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

The sample mean is a random variable before data are collected. For n independent observations from the same population with mean μ and finite variance σ^2, its mean is μ and its standard deviation, or standard error, is $\sigma/\sqrt{n}$.

For a normal population the sample mean is normal for every n. For many other populations it becomes approximately normal as n grows, under the central limit theorem. The adequacy of an approximation depends on skewness, tails, independence and sample size.

Worked example

A population has mean 40 and SD 12. For independent random samples of size 36, find the mean and SD of the sample mean.

The mean remains μ

standard error is $12/\sqrt{36}=2$.

Answer: Mean 40; SD 2.

Try it yourself

Name: _____ Date: _____

1. A population has mean 70 and SD 10. Find the mean and variance of the mean of 25 independent observations.

2. A normal population is N(100, 64). State the exact distribution of the mean of 16 independent observations.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. Mean 70; variance 4.

Use $\text{Var}(\bar{X})=\sigma^2/n=100/25$.

2. N(100, 4).

The mean stays 100 and variance is 64/16.

Curriculum reference: Specialist Mathematics Units 3–4 — Distribution of the sample mean

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