Vector motion, paths and collisions

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Show the mathematical steps. A calculator, graphing tool or CAS may be used to check results where useful.

What you will practise

• Vector calculus and motion in two and three dimensions

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

Differentiate or integrate a vector function component by component. Velocity is $r'(t)$, acceleration is $r''(t)$, and speed is $|r'(t)|$. An initial position supplies the integration constants needed to recover a position vector from velocity.

Paths cross if two particles visit the same location, possibly at different times. A collision requires the same location at the same time. When describing a geometric path, preserve the time or parameter interval and distinguish position from distance travelled.

Worked example

For $r(t)=(t^2, 3t, 2)$, find velocity, acceleration and speed at $t=1$.

Differentiate each component

take the magnitude of $v(1)=(2, 3, 0)$.

Answer: $v=(2t, 3, 0)$, $a=(2, 0, 0)$; speed $\sqrt{13}$ at $t=1$.

Try it yourself

Name: _____ Date: _____

1. For r(t)=(cos t, sin t), find the speed.

2. Integrate v(t)=(2 t, 1, -3) given r(0)=(1, 2, 4).

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. 1.

$v=(-\sin t, \cos t)$, whose magnitude is $\sqrt{(\sin^2 t+\cos^2 t)}=1$.

2. $r(t)=(t^2+1, t+2, 4-3t)$.

Integrate each component and use the three initial coordinates.

Curriculum reference: Specialist Mathematics Units 3–4 — Vector calculus and motion in two and three dimensions

Independent original resource. Not published or endorsed by VCAA. This focused set supports the listed skills; it is not a complete course or an official assessment.

This free preview may be shared intact with attribution. Not for sale.