Three-dimensional vectors and geometric proof

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A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the worked example, then show each step in the practice questions. Attempt the reasoning and exact calculations by hand. Technology may check a result but cannot replace the written argument.

What you will practise

• Vectors, linear dependence, dot product, cross product and geometric proof

Useful starting skills: Specialist and Methods Units 1–2; concurrent or completed Mathematical Methods Units 3–4.

A three-dimensional vector can be represented by its components in orthogonal unit directions. The dot product measures alignment and gives projections; the cross product is perpendicular to both input vectors and has magnitude equal to their spanned parallelogram area.

Vectors are linearly dependent when a nontrivial linear combination equals zero. Use position vectors to convert geometric statements into algebra, and state any equal-length or perpendicularity assumptions used in the proof.

Worked example

For $a=(1, 2, 2)$ and $b=(2, 0, 1)$, find $a\cdot b$ and the scalar projection of a onto b.

The dot product is $1\times2+2\times0+2\times1=4$

divide by $|b|=\sqrt{5}$.

Answer: $a\cdot b=4$; scalar projection $4/\sqrt{5}$.

Try it yourself

Name: _____ Date: _____

1. Find the magnitude and a unit vector in the direction of (2, -1, 2).

2. Find (1, 0, 0)×(0, 1, 0) and reverse the order.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. Magnitude 3; unit vector (2/3, -1/3, 2/3).

Compute √(4+1+4), then divide every component by 3.

2. (0, 0, 1); reversed product (0, 0, -1).

Use the orientation of the standard basis; reversing the order negates a cross product.

Curriculum reference: Specialist Mathematics Units 3–4 — Vectors, linear dependence, dot product, cross product and geometric proof

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