Rates of change and limits

Learn and practise | Free preview

A sample from this topic. The complete topic collection contains three different resources and worked solutions.

Read the explanation and worked example. Show your working; use a calculator only where asked.

What you will practise

• Compare average rates of change and investigate a limiting instantaneous rate.

• Explain a method and check the reasonableness of a result.

Useful starting skills: Use the prerequisite operations and representations shown in the worked example.

An average rate of change is the change in output divided by the corresponding non-zero change in input. On a graph it is the gradient of a secant line between two points; its units are output units per input unit.

A sequence of smaller intervals can suggest a limiting rate at a point. For $y=x^2$, the rate from $x=a$ to $x=a+h$ is $2a+h$ when $h\neq0$. As h approaches zero this approaches $2a$. The original quotient is undefined at $h=0$, so investigate the limit instead of substituting zero into its denominator.

Worked example

Find the average rate for $y=x^2$ between $x=3$ and $x=3.1$, then predict the limiting rate at $x=3$.

The output change is $3.1^2-3^2=0.61$.

Divide by 0.1 to get 6.1. In general the rate is $6+h$.

As h approaches 0, the rate approaches 6.

Answer: Average rate 6.1; limiting rate 6

Try it yourself

Name: _____ Date: _____

1. For $y=x^2$, find the average rate from $x=1$ to $x=3$.

2. A cyclist’s cumulative distance travelled is 12 km at $t=0.5$ h and 30 km at $t=1.25$ h. Find the average speed over this interval.

Answers and explanations

Check the method as well as the final answer. An alternative correct method is also valid.

1. 4

(9-1)/(3-1)=4.

2. 24 km/h

Divide the 18 km distance change by 0.75 h.

Curriculum reference: VC2M10AM02

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